Connection Between the Relativistic and Nonrelativistic Form of the Nucleon-Nucleon Amplitude*

A. O. BARUT AND M. SAMIULLAH
Department of Physics, University of Colorado, Boulder, Colorado
(Received 8 May 1964)

The connection between the nonrelativistic (Wolfenstein-Ashkin) and the relativistic (Goldberger-Grisaru-MacDonald-Wong) amplitudes of the nucleon-nucleon scattering is obtained using a two-component form of the latter. This connection involves simple functions of E and $\cos\theta$. It is then possible to translate the analysis of scattering and polarization experiments in terms of one set of amplitudes into the other.

THE most general form of the scattering amplitude is expressed as a linear combination of well-defined spinor basis functions invariant under rotations or Lorentz transformations, respectively, multiplied with scalar coefficients which contain the dynamics. Because both the nonrelativistic and the relativistic form of the scattering amplitude has been used extensively in the analysis of nucleon-nucleon scattering and polarization experiments it is of interest to give explicitly their connection. It is then possible to translate one type of analysis into another.

Under the invariance with respect to three-dimensional rotations, parity and time-reversal transformations the most general form of the baryon-baryon scattering amplitude is given by[1]

$$R=A+B(\sigma_{1n}+\sigma_{2n})+C(\sigma_{1n}\sigma_{2n}-1)+D(\sigma_{1n}-\sigma_{2n}) +E\sigma_{1K}\sigma_{2K}+F\sigma_{1P}\sigma_{2P}, \quad (1)$$

where

$$\sigma_{in}=\boldsymbol{\sigma}_i\cdot\mathbf{n}, \quad \sigma_{iK}=\boldsymbol{\sigma}_i\cdot\mathbf{K}, \quad \sigma_{iP}=\boldsymbol{\sigma}_i\cdot\mathbf{P},$$

with

$$\mathbf{n}=\mathbf{q}'\times\mathbf{q}, \quad \mathbf{K}=\mathbf{q}'-\mathbf{q}, \quad \mathbf{P}=\mathbf{n}\times\mathbf{K}.$$

Here $\mathbf{q}'$ and $\mathbf{q}$ are the center-of-mass momenta in the final and initial states, respectively, and the coefficients A to F are functions of energy and scattering angle which are invariant under rotations but not under Lorentz transformations. The term D $(\sigma_{1n}-\sigma_{2n})$ drops out for identical particles, for example, for nucleon-nucleon scattering under charge independence in which case we have five amplitudes for each value of the total isotopic spin.

A convenient covariant set of amplitudes for the nucleon-nucleon problem has been given by Goldberger *et al.*[2] The connection between the WA amplitudes (1) and the GGMW amplitudes is most easily established if we write the covariant amplitudes in the two-component formalism[3]

$$R=\sum_{i=1}^{5} A_i(Y_i+(-1)^i\tilde{Y}_i), \quad (2)$$

where A_i are the Lorentz scalar amplitudes, functions of the invariant scalar products of the momenta,

$$s=(k_1+k_2)^2, \quad t=(k_1-k_3)^2, \quad u=(k_1-k_4)^2; \quad s+t+u=4,$$

k_1 and k_2 being the *physical* energy momentum four vectors of the two incoming particles and k_3 and k_4 those of the two outgoing particles. We use units such that nucleon mass is unity. The spin basis Y_i are given by

$$\begin{aligned}
Y_1&=[(k_4\cdot\sigma k_2\cdot\tilde{\sigma})^{1/2}+(k_4\cdot\tilde{\sigma}k_2\cdot\sigma)^{1/2}] \otimes[(k_3\cdot\sigma k_1\cdot\tilde{\sigma})^{1/2}+(k_3\cdot\tilde{\sigma}k_1\cdot\sigma)^{1/2}],\\
Y_2&=-\tfrac{1}{4}[(k_4\cdot\sigma)^{1/2}(\tilde{\sigma}^\mu\sigma^\nu-\tilde{\sigma}^\nu\sigma^\mu)(k_2\cdot\tilde{\sigma})^{1/2} +(k_4\cdot\tilde{\sigma})^{1/2}(\sigma^\mu\tilde{\sigma}^\nu-\sigma^\nu\tilde{\sigma}^\mu)(k_2\cdot\sigma)^{1/2}]\\
&\quad\otimes[(k_3\cdot\sigma)^{1/2}(\tilde{\sigma}_\mu\sigma_\nu-\tilde{\sigma}_\nu\sigma_\mu)(k_1\cdot\tilde{\sigma})^{1/2} +(k_3\cdot\tilde{\sigma})^{1/2}(\sigma_\mu\tilde{\sigma}_\nu-\sigma_\nu\tilde{\sigma}_\mu)(k_1\cdot\sigma)^{1/2}],\\
Y_3&=-[(k_4\cdot\sigma)^{1/2}\tilde{\sigma}^\mu(k_2\cdot\sigma)^{1/2}-(k_4\cdot\tilde{\sigma})^{1/2}\sigma^\mu(k_2\cdot\tilde{\sigma})^{1/2}] \otimes[(k_3\cdot\sigma)^{1/2}\tilde{\sigma}_\mu(k_1\cdot\sigma)^{1/2}-(k_3\cdot\tilde{\sigma})^{1/2}\sigma_\mu(k_1\cdot\tilde{\sigma})^{1/2}],\\
Y_4&=[(k_4\cdot\sigma)^{1/2}\tilde{\sigma}^\mu(k_2\cdot\sigma)^{1/2}+(k_4\cdot\tilde{\sigma})^{1/2}\sigma^\mu(k_2\cdot\tilde{\sigma})^{1/2}] \otimes[(k_3\cdot\sigma)^{1/2}\tilde{\sigma}_\mu(k_1\cdot\sigma)^{1/2}+(k_3\cdot\tilde{\sigma})^{1/2}\sigma_\mu(k_1\cdot\tilde{\sigma})^{1/2}],\\
Y_5&=[(k_4\cdot\sigma k_2\cdot\tilde{\sigma})^{1/2}-(k_4\cdot\tilde{\sigma}k_2\cdot\sigma)^{1/2}] \otimes[(k_3\cdot\sigma k_1\cdot\tilde{\sigma})^{1/2}-(k_3\cdot\tilde{\sigma}k_1\cdot\sigma)^{1/2}]. \quad (3)
\end{aligned}$$

Here we use covariant Pauli matrices

$$\sigma^\mu\equiv(I,-\boldsymbol{\sigma}), \quad \sigma_\mu\equiv(I,\boldsymbol{\sigma}), \quad \tilde{\sigma}_\mu\equiv(I,-\sigma) \quad (4)$$

and the notation $k\cdot\sigma=k^\mu\sigma_\mu$.[4]

The spin basis $\tilde{Y}_i$ is obtained from Y_i given in Eq. (3) by the interchange of k_3 and k_4 so that the two systems are related to each other by the Fierz matrix

$$\tilde{Y}_i=\sum_j \Gamma_{ij}Y_j,$$

where

$$\Gamma=\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 & 1\\ 6 & -2 & 0 & 0 & 6\\ 4 & 0 & -2 & 2 & -4\\ 4 & 0 & 2 & -2 & -4\\ 1 & 1 & -1 & -1 & 1 \end{pmatrix}.$$

* Supported in part by the U. S. Air Force Office of Scientific Research and the National Science Foundation.

[1] L. Wolfenstein and J. Ashkin, Phys. Rev. 85, 947 (1952). Referred as WA.

[2] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960), referred to hereafter as GGMW. Our A_i are the same amplitudes as the amplitudes F_i of GGMW.

[3] A. O. Barut and B. C. Unal (to be published).

[4] For the use of two-component spinors and spinor indices of the amplitudes see Appendix I of A. O. Barut, I. Muzinich, and D. N. Williams, Phys. Rev. 130, 442 (1963). The first factor of the direct product refers always to particles 2 and 4 and the spinor indices of this factor represents the spin components of these particles; the second factor similarly refer to particles 1 and 3.

Equation (2) becomes then

$$R=\sum A_i(I+(-1)^i\Gamma)_{ij}Y_j. \quad (5)$$

With this choice of the basis the Pauli principle can be expressed simply by the following symmetry relation on the scalar amplitudes

$$A_i^I(s,t,u)=(-1)^{i+I}A_i^I(s,u,t), \quad (6)$$

where I is the total isotopic spin.

With the help of formula given in the Appendix we expand the square roots in Eq. (3) and obtain in the center-of-mass frame

$$\begin{aligned}Y_1=4[&a^4\delta\otimes\delta-a^2b^2(\delta\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\delta)\\&+b^4(\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma})],\\
Y_2=-4[&a^4\sigma^i\otimes\sigma_i-a^2b^2(\sigma^i\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\sigma_i\\&+\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i-\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i\\&-\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\otimes\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma})+b^4\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma}],\\
Y_3=4[&a^2b^2(\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}+\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\\&+\mathbf{q}'\cdot\boldsymbol{\sigma}\otimes\mathbf{q}\cdot\boldsymbol{\sigma}-\sigma^i\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma}-\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\sigma_i)\\&-a^4\sigma^i\otimes\sigma_i-b^4(\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma})],\\
Y_4=4[&a^2b^2(\delta\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\delta-\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma}\\&-\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i-\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i-\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\otimes\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma})\\&+a^4\delta\otimes\delta+b^4(\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma})],\\
Y_5=-4&a^2b^2(\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\\&-\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}-\mathbf{q}'\cdot\boldsymbol{\sigma}\otimes\mathbf{q}\cdot\boldsymbol{\sigma}), \quad (7)\end{aligned}$$

where

$$a=(1/\sqrt{2})(E+1)^{1/2}, \quad b=(1/\sqrt{2}q)(E-1)^{1/2}. \quad (8)$$

If we insert Eqs. (7) into (5) and collect terms together we can write the R amplitude in the form

$$\begin{aligned}R=A\,\delta\otimes\delta+B(\delta\otimes\boldsymbol{\sigma}+\boldsymbol{\sigma}\otimes\delta)\cdot\mathbf{n}+C(\sigma_i\otimes\sigma_i-1)\\+E\boldsymbol{\sigma}\cdot\mathbf{K}\otimes\boldsymbol{\sigma}\cdot\mathbf{K}+F\boldsymbol{\sigma}\cdot\mathbf{P}\otimes\boldsymbol{\sigma}\cdot\mathbf{P}, \quad (9)\end{aligned}$$

which is exactly of the type of Eq. (1) and we find for the coefficients

$$\begin{aligned}A=&-4A_1(E^2-1)\cos\theta+4A_2[(E-1)^2\cos^2\theta+2E]\\&-4A_3(E^2-1)\cos\theta+4A_4[(E-1)^2\cos^2\theta+2E],\\
B=&-2A_1(E^2-1)+A_2[2(E-1)^2\cos\theta-(E^2-1)]\\&-3A_3(E^2-1)+A_4[2(E-1)^2\cos\theta+(E^2-1)]\\&+A_5(E^2-1),\\
C=&-A_1[(E^2-1)\cos\theta+(E^2+1)]\\&+2A_2[(E-1)^2\cos^2\theta+(E^2-1)\cos\theta+(2E-1)]\\&-2A_3[2(E^2-1)\cos\theta-(2E^2+1)]\\&+2A_4[(E-1)^2\cos^2\theta-E(E-2)]\\&-A_5[(E^2-1)\cos\theta+(E^2-1)],\\
E=&A_2[(E-1)^2(1+\cos\theta)-(E^2-1)]-3A_3(E^2-1)\\&+A_4[(E-1)^2(1+\cos\theta)+(E^2-1)]-A_5(E^2-1),\\
(1-\cos\theta)^2F=&A_2[(E^2-1)(1-\cos\theta)-(E^2-1)]\\&+3A_3(E^2-1)+A_4[(E-1)^2(1-\cos\theta)+(E^2-1)]\\&+A_5(E^2-1). \quad (10)\end{aligned}$$

These are the desired relations between the nonrelativistic amplitudes A, B, C, E, and F and the relativistic amplitudes A_1 to A_5. The coefficients involve only E and $\cos\theta$, both invariant quantities under rotations.

[*Note added in proof*. Because the scalar amplitudes A_i in (2) are free of kinematical singularities, they can be considered constant at threshold; Eq. (10) gives then threshold behavior of the full amplitude.]

The evaluation of certain quantities are much easier in the form (1) than (5). For example, for the polarization of one outgoing nucleon

$$P_3=\mathrm{tr}(R^\dagger\sigma_3R)/\mathrm{tr}(R^\dagger R), \quad (11)$$

we obtain

$$\begin{aligned}P_3=2n_3\,\mathrm{Re}A^*C/[&|A-B|^2+3|B|^2\\&+\sin^2\theta(|C|^2+8\sin^2\tfrac{1}{2}\theta\,\mathrm{Re}B^*F)\\&+16\sin^4\tfrac{1}{2}\theta(|E|^2+|F|^2\sin^4\theta+\tfrac{1}{2}\,\mathrm{Re}B^*E\csc^2\tfrac{1}{2}\theta)]\end{aligned}$$

which can be easily rewritten in terms of the relativistic amplitudes by substituting the Eqs. (10).

APPENDIX

The covariant Pauli matrices satisfy the relations

$$\sigma^\mu\tilde{\sigma}^\nu=g^{\mu\nu}+\tfrac{1}{2}i\epsilon^{\mu\nu\lambda\rho}\sigma_\lambda\tilde{\sigma}_\rho,$$
$$\tilde{\sigma}^\mu\sigma^\nu=g^{\mu\nu}-\tfrac{1}{2}i\epsilon^{\mu\nu\lambda\rho}\tilde{\sigma}_\lambda\sigma_\rho.$$

The Hermitian square roots $(k\cdot\sigma)^{1/2}$ can be expanded as follows

$$(k^\mu\sigma_\mu)^{1/2}=a+b\boldsymbol{\sigma}\cdot\mathbf{q},$$
$$(k^\mu\tilde{\sigma}_\mu)^{1/2}=a-b\boldsymbol{\sigma}\cdot\mathbf{q},$$

where the coefficients a and b are given in the text by Eq. (8). For the direct product of tensors of Pauli matrices and momenta we have used the following identities:

$$\mathrm{tr}(\sigma_j\sigma_k)=2\delta_{jk};\ \mathrm{tr}(\sigma_j\sigma_k\sigma_l)=2i\epsilon_{jkl},$$

(1) $\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}=\mathbf{q}'\cdot\mathbf{q}+i(\mathbf{q}'x\mathbf{q})\cdot\boldsymbol{\sigma}$,

(2) $\delta\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\delta=2\mathbf{q}'\cdot\mathbf{q}\delta\otimes\delta+i(\delta\otimes\boldsymbol{\sigma}+\boldsymbol{\sigma}\otimes\delta)\cdot\mathbf{n}$,

(3) $\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\mathbf{q}\cdot\boldsymbol{\sigma}=(\mathbf{q}'\cdot\mathbf{q})^2\delta\otimes\delta+\mathbf{q}'\cdot\mathbf{q}i(\delta\otimes\boldsymbol{\sigma}+\boldsymbol{\sigma}\otimes\delta)\cdot\mathbf{n}-\mathbf{n}\cdot\boldsymbol{\sigma}\otimes\mathbf{n}\cdot\boldsymbol{\sigma}$,

(4) $\sigma^i\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\sigma_i\mathbf{q}\cdot\boldsymbol{\sigma}+\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma^i\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}\sigma_i=-2\mathbf{q}^2(\delta\otimes\delta+\sigma^i\otimes\sigma_i)-\mathbf{q}\cdot\boldsymbol{\sigma}\otimes\mathbf{q}\cdot\boldsymbol{\sigma}-\mathbf{q}'\cdot\boldsymbol{\sigma}\otimes\mathbf{q}'\cdot\boldsymbol{\sigma}-2i(\delta\otimes\boldsymbol{\sigma}+\boldsymbol{\sigma}\otimes\delta)\cdot\mathbf{n}$,

(5) $\mathbf{q}'\cdot\sigma\sigma^i\otimes\sigma_i\mathbf{q}\cdot\sigma+\sigma^i\mathbf{q}\cdot\sigma\otimes\mathbf{q}'\cdot\sigma\sigma_i=-2\mathbf{q}'\cdot\mathbf{q}(\delta\otimes\delta-\sigma^i\otimes\sigma_i)+\mathbf{q}\cdot\sigma\otimes\mathbf{q}'\cdot\sigma+\mathbf{q}'\cdot\sigma\otimes\mathbf{q}\cdot\sigma-2i(\delta\otimes\sigma+\sigma\otimes\delta)\cdot\mathbf{n}$,

(6) $\sigma^i\otimes\mathbf{q}'\cdot\sigma\sigma_i\mathbf{q}\cdot\sigma+\mathbf{q}'\cdot\sigma\sigma^i\mathbf{q}\cdot\sigma\otimes\sigma_i=-2\mathbf{q}\cdot\sigma\otimes\mathbf{q}'\cdot\sigma-2\mathbf{q}'\cdot\sigma\otimes\mathbf{q}\cdot\sigma-2\mathbf{q}'\cdot\mathbf{q}\sigma^i\otimes\sigma_i+i(\delta\otimes\sigma+\sigma\otimes\delta)\cdot\mathbf{n}$,

(7) $\mathbf{q}'\cdot\sigma\sigma^i\mathbf{q}\cdot\sigma\otimes\mathbf{q}'\cdot\sigma\sigma_i\mathbf{q}\cdot\sigma=-q^2(\mathbf{q}'\cdot\sigma\otimes\mathbf{q}'\cdot\sigma+\mathbf{q}\cdot\sigma\otimes\mathbf{q}\cdot\sigma)+\mathbf{q}'\cdot\mathbf{q}(\mathbf{q}\cdot\sigma\otimes\mathbf{q}'\cdot\sigma+\mathbf{q}'\cdot\sigma\otimes\mathbf{q}\cdot\sigma)+\delta\otimes\delta q^4+i\mathbf{q}'\cdot\mathbf{q}$
$\times(\sigma\otimes\delta+\delta\otimes\sigma)\cdot n-(\mathbf{q}'\cdot\mathbf{q})^2\delta\otimes\delta+(\mathbf{q}'\cdot\mathbf{q})^2\sigma^i\otimes\sigma_i$,

(8) $$\mathbf{q}'\cdot\sigma\otimes\mathbf{q}'\cdot\sigma+\mathbf{q}\cdot\sigma\otimes\mathbf{q}\cdot\sigma=\frac{1}{2}\left[\frac{\sigma\cdot\mathbf{P}\otimes\sigma\cdot\mathbf{P}}{(q^2-\mathbf{q}'\cdot\mathbf{q})^2}+\sigma\cdot\mathbf{K}\otimes\sigma\cdot\mathbf{K}\right],$$

(9) $$\mathbf{q}'\cdot\sigma\otimes\mathbf{q}\cdot\sigma+\mathbf{q}\cdot\sigma\otimes\mathbf{q}'\cdot\sigma=\frac{1}{2}\left[\frac{\sigma\cdot\mathbf{P}\otimes\sigma\cdot\mathbf{P}}{(q^2-\mathbf{q}'\cdot\mathbf{q})^2}-\sigma\cdot\mathbf{K}\otimes\sigma\cdot\mathbf{K}\right],$$

(10) $$\sigma\cdot\mathbf{n}\otimes\sigma\cdot\mathbf{n}=-[q^4-(\mathbf{q}'\cdot\mathbf{q})^2]\sigma^i\otimes\sigma_i-\frac{1}{2}\frac{\sigma\cdot\mathbf{P}\otimes\sigma\cdot\mathbf{P}}{(q^2-\mathbf{q}'\cdot\mathbf{q})}-\tfrac{1}{2}(q^2+\mathbf{q}'\cdot\mathbf{q})\sigma\cdot\mathbf{K}\otimes\sigma\cdot\mathbf{K}.$$

PHYSICAL REVIEW VOLUME 135, NUMBER 6B 21 SEPTEMBER 1964

Breaking of SU(3) Symmetry in the $\frac{3}{2}^+$ Meson-Baryon Decuplet

K. C. WALI*
Argonne National Laboratory, Argonne, Illinois

AND

ROBERT L. WARNOCK
Illinois Institute of Technology, Chicago, Illinois

(Received 14 May 1964)

Breaking of SU(3) invariance is studied in the case of the meson-baryon decuplet with $J^P=\frac{3}{2}^+$. In a simple theory based on single-baryon exchange, the evolution of the broken decuplet from a degenerate origin is traced in detail. Symmetry breaking is introduced by allowing initially degenerate meson and baryon masses to approach continuously their physical values, while obeying at every stage the Gell-Mann–Okubo sum rules. The following conclusions are reached: (i) The Okubo equal spacing rule for the decuplet levels follows from the validity of the Gell-Mann–Okubo rules for the meson and baryon octuplets. (ii) The main effect of the symmetry breaking may be characterized as a mixing of the 10- and 27-dimensional representations. The mixing is small enough so that the resonances can be unambiguously associated with the 10 representation, but at the same time large enough to imply coupling-constant ratios differing appreciably from the values for pure symmetry. (iii) In the approach to pure symmetry through reduction of mass differences, there are no difficulties of the type pointed out by Oakes and Yang. Resonances cross thresholds smoothly, and a degenerate decuplet of bound states is obtained in the limit.

1. INTRODUCTION

MESON-BARYON resonances with $J^P=\frac{3}{2}^+$ have been assigned tentatively[1] to the (3,0) decuplet representation of the group SU(3).[2] The resonant states in question are $N_{3/2}^*$ (1238 MeV, $T=\frac{3}{2}$, $Y=1$), Y_1^* (1385 MeV, $T=1$, $Y=0$), $\Xi_{1/2}^*$ (1530 MeV, $T=\frac{1}{2}$, $Y=-1$). It now appears likely that these states all have the correct $\frac{3}{2}^+$ spin-parity values.[3] To complete the decuplet, a particle Ω_0 (sometimes called Ω_-) with $T=0$, $Y=-2$ was predicted. The recent discovery[4] of such a particle constitutes strong evidence for both the decuplet assignment and the general scheme of the "eightfold way."[2] The discovery is all the more remarkable, since the observed mass (1686±12 MeV) of Ω_0 agrees very well with the prediction of the Gell-Mann–Okubo mass formula.[2,5] For the (3,0) decuplet, the

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. **34**, 1 (1962); S. Glashow and J. J. Sakurai, Nuovo Cimento **26**, 622 (1962); M. Gell-Mann, *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962); R. Cutkosky, J. Kalckar, and P. Tarjanne, Phys. Letters **1**, 93 (1962); R. H. Capps, Nuovo Cimento **27**, 1208 (1963).

[2] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[3] A summary of the experiments and a bibliography is given by R. H. Dalitz, Ann. Rev. Nucl. Sci. **13**, 339 (1963).

[4] V. E. Barnes, P. L. Connolly, D. J. Grennell, B. B. Culwick, *et al.*, Phys. Rev. Letters **12**, 204 (1964). We prefer the notation Ω_0 to Ω_-, since the isotopic spin has been used as a subscript for the other decuplet states. The Ω_0 notation appears also in Ref. 7.

[5] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).